Math 2 HW # 1
Key

1a $\binom{52}{5} = \dfrac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

1b Four of a kind: # of ways:

13 choices for the rank of the four matching cards

$52 - 4 = 48$ choices for remaining card

$\Rightarrow$ $13 \cdot 48$ possible hands

Probability $= \dfrac{13 \cdot 48}{\binom{52}{5}} \approx 0.00024$

1c At least a pair I think the easiest way to count these is to subtract off the number of hands with no pairs:

$\binom{13}{5}$ ways to choose 5 distinct card ranks

$\times 4 \times 4 \times 4 \times 4 \times 4$ ways to assign a suit to each card.

So, there are $\binom{52}{5} - \binom{13}{5} 4^5$ hands with no pairs; the prob. of this event is

$$\frac{\binom{52}{5} - \binom{13}{5} 4^5}{\binom{52}{5}} \approx 0.493$$

1d **Flush** Number of possible hands =

4 choices of suit

$\times \binom{13}{5}$ choices of 5 cards from the suit.

$$\text{Prob.} = \frac{4\binom{13}{5}}{\binom{52}{5}} \approx 0.00198$$

2 After the elk are tagged, there are 5 with tags
15 without tags

If you capture four of these, there are $\binom{5}{2}\binom{15}{2}$ ways to choose 2 with tags and 2 without

Assume the elk have mixed around so that each choice of four elk is equally likely. The prob. that exactly 2 of the four are tagged is

$$\binom{5}{2}\binom{15}{2} \Big/ \binom{20}{4} \approx 0.217$$

Some students interpret the question as "at least 2 tagged". In this case the answer is

$$\frac{\binom{5}{2}\binom{15}{2}+\binom{5}{3}\binom{15}{1}+\binom{5}{4}\binom{15}{0}}{\binom{20}{4}} \approx 0.249$$

3 Give names to the following events:

F	The fair coin is chosen
D	the two-headed coin chosen
W	The weighted trick coin chosen

And events for the coin flips:

H	The flip is heads
T	The flip is tails.

we assume that

$$P(F) = P(D) = P(W) = 1/3.$$

What we know about the coins is

$$P(H \mid F) = 1/2$$
$$P(H \mid W) = 3/4$$
$$P(H \mid D) = 1.$$

Must calculate $P(D \mid H)$.

$$P(D \mid H) = \frac{P(D \cap H)}{P(H)}$$

Since $D \subset H$, $P(D \cap H) = P(D) = 1/3$

Calc. $P(H)$ by conditioning:

$$P(H) = P(H \mid F)P(F) + P(H \mid W)P(W) + P(H \mid D)P(D)$$

$$= 1/2 \cdot 1/3 + 3/4 \cdot 1/3 + 1 \cdot 1/3$$

$$= 9/12$$

$$\Longrightarrow P(D \mid H) = \frac{1/3}{9/12} = \frac{4}{9}$$

4 Events: S = slice

N = No slice

$4I$ = used 4-iron

$5I$ = used 5-iron

$5W$ = used 5-wood

We are given that

$P(4I) = .4$, $P(5I) = .4$, $P(5W) = .2$

$P(S|4I) = .5$, $P(S|5I) = .3$, $P(S|5W) = .55$

Must calculate $P(4I|S)$.

$$P(4I|S) = \frac{P(4I \cap S)}{P(S)}$$

First the numerator:

$$P(4I \cap S) = P(S|4I)P(4I)$$
$$= .5 \times .4 = .2$$

Now the denominator:

$$P(S) = P(S|4I)P(4I) + P(S|5I)P(5I) + P(S|5W)P(5W)$$
$$= .5 \times .4 + .4 \times .3 + .2 \times .55$$
$$= .43$$

Answer: $P(4I|S) = \frac{.2}{.43} \approx .465$

5 We should model this RV as Poisson with mean 3.1 because there are a large number of leaves, each with a small and approximately independent chance of falling during any minute.

Poisson(λ): $P(X=k) = e^{-\lambda} \cdot \lambda^k / k!$

Therefore $P(X=5) = e^{-3.1} \cdot (3.1)^5 / 5!$

≈ 0.107

Let Y be # of leaves falling in 3 minutes.

Then $Y \stackrel{d}{=} \text{Poisson}(3 \times 3.1)$.

$P(Y \leq 1) = e^{-9.3} (9.3)^0 / 0! + e^{-9.3} (9.3)^1 / 1!$

≈ 0.00094

(wow, very unlikely.)

6 Let the dice rolls be X_1, X_2.

We know that the expected value of a dice roll is 3.5. The variance of a dice roll is

$$E[(X_1 - 3.5)^2] = \tfrac{1}{6}(1-3.5)^2 + \tfrac{1}{6}(2-3.5)^2 + \tfrac{1}{6}(3-3.5)^2 + \tfrac{1}{6}(4-3.5)^2 + \tfrac{1}{6}(5-3.5)^2 + \tfrac{1}{6}(6-3.5)^2$$

$$= 35/12$$

$E[X] = E[X_1 + X_2] = E[X_1] + E[X_2] = 7$

Since X_1 & X_2 indep, their variances add:

$Var[X_1 + X_2] = Var[X_1] + Var[X_2] = 35/6$

Thus $E[X] = 7$, $Var[X] = \frac{35}{6}$, $\sigma[X] = \sqrt{\frac{35}{6}}$.